# PARAMETERIZING SINGLE SCATTERING PROPERTIES ACROSS THE ELECTROMAGNETIC SPECTRUM FOR WATER CLOUD RETRIEVAL

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### Abstract

Water clouds are composed of spherical water droplets, to which the Mie solution applies for their electromagnetic scattering. The essential variable of Mie solutions is $\xi \equiv mkD$ (or equivalently $mkr$), where $m$ is the complex index of refraction of water, $k = 2\pi/\lambda$ the angular wavenumber, $\lambda$ the wavelength, and $D$ ($r$) the diameter (radius) of the water droplet. Consequently, all water-droplet single-scattering properties, such as extinction and scattering efficiencies, i.e., $Q_e$ and $Q_s$, are functions of $\xi$, which provides a convenient pathway to parameterize water-droplet single-scattering properties across the electromagnetic spectrum or spectrum segments. In this paper, we first demonstrate that the numerical Mie solution indeed depends only on $\xi$. We then present our first attempt to parameterize extinction efficiency across the visible-infrared spectrum.

***Index Terms*—** One, two, three, four, five

## 1. INTRODUCTION

As stated in [1], "The simultaneous retrieval of cloud optical thickness and effective radius is best achieved by simultaneously measuring the reflection function in a non-absorbing and absorbing spectral channel (e.g., VIS/NIR and SWIR, respectively), and comparing the resulting measurements with theoretical forward model calculations". However, with a multispectral (or hyperspectral) instrument, there are multiple (or many) options for both the "non-absorbing" and "absorbing" spectral channels to choose from. Ideally, the (water) cloud optical thickness (COT) and cloud effective radius (CER) retrieved using different channel combinations should be consistent. This consistency is easier to evaluate if a functional relation exists that expresses the spectral dependence of COT on frequency (wavelength).

Since the droplets in water clouds are considered spherical and the applicable Mie solution for their electromagnetic scattering depends solely on $\xi \equiv mkD$, the single-scattering quantities of these droplets, such as the extinction and scattering efficiencies ($Q_e$ and $Q_s$ respectively), are functions of $\xi$ only. This variable, $\xi$, thus provides a convenient pathway for parameterizing these scattering quantities into functional relations across the electromagnetic spectrum or, at least, spectrum segments.

In the following we derive first the equations to be used for the intended parameterization. Next, we provide numerical evidences attesting to the $\xi$-only dependence. We then present the results from our first attempt at the parameterization of $Q_e$. Finally, we conclude.

## 2. THEORETICAL DERIVATION

Optical thickness $\tau$ is defined as

$$d\tau = \beta_e dz$$

where $dz$ is the differential height or thickness of a plane-parallel cloud layer, $\beta_e$ is the extinction coefficient (with a unit of inverse distance, e.g. m$^{-1}$). The extinction coefficient, in turn, is the integral of the product of extinction cross section $\sigma_e = \pi r^2 Q_e(\xi)$ and the droplet size distribution (DSD) within the differential cloud layer $dz$, i.e.,

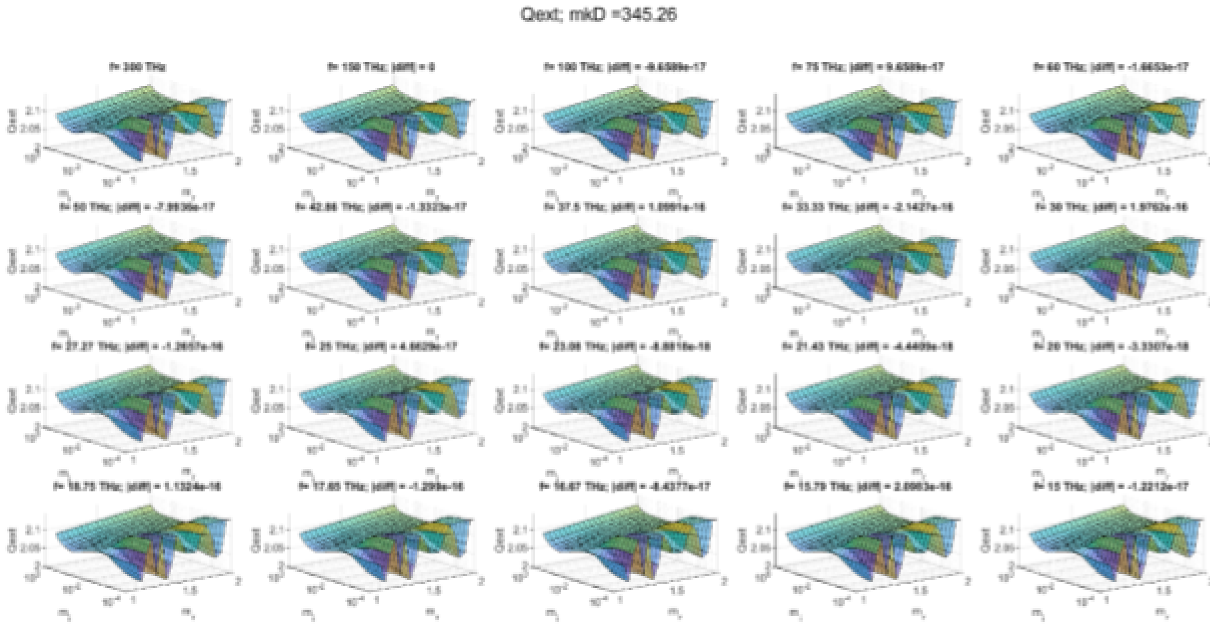


Fig. 1 $Q_e$ as a function of $(m_r, m_i)$ at a constant $mkD$=345.26 for a fixed set of $f$.

$$\beta_e = \int \sigma_e(r)n(r)dr = \pi \int Q_e(\xi)r^2 n(r)dr$$

where $n(r)dr$ is the droplet size distribution within the differential cloud layer. We note that

$$\xi = mkD = 2mkr,$$

in which the complex index of refraction $m$ depends on both the wavelength (frequency) and the dielectric material. In our study, the material is fixed, i.e., liquid water. Therefore, both $m$ and $k = 2\pi/\lambda = 2\pi c/f$, where $c$ is the speed of light, are functions of wavelength (frequency) $\lambda$ $(f)$ only. Similarly, $Q_e(\xi) \equiv Q_e(\lambda, r) \equiv Q_e(f, r)$ and, in turn,

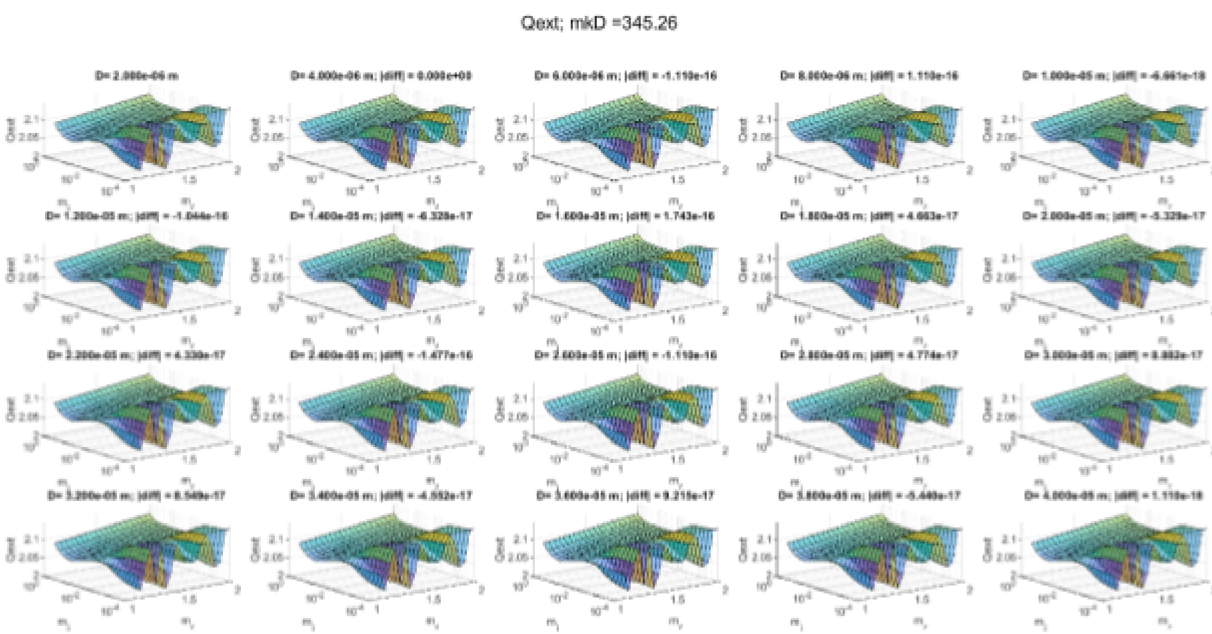


Fig. 2 $Q_e$ as a function of $(m_r, m_i)$ at a constant $mkD$=345.26 for a fixed set of $D$.

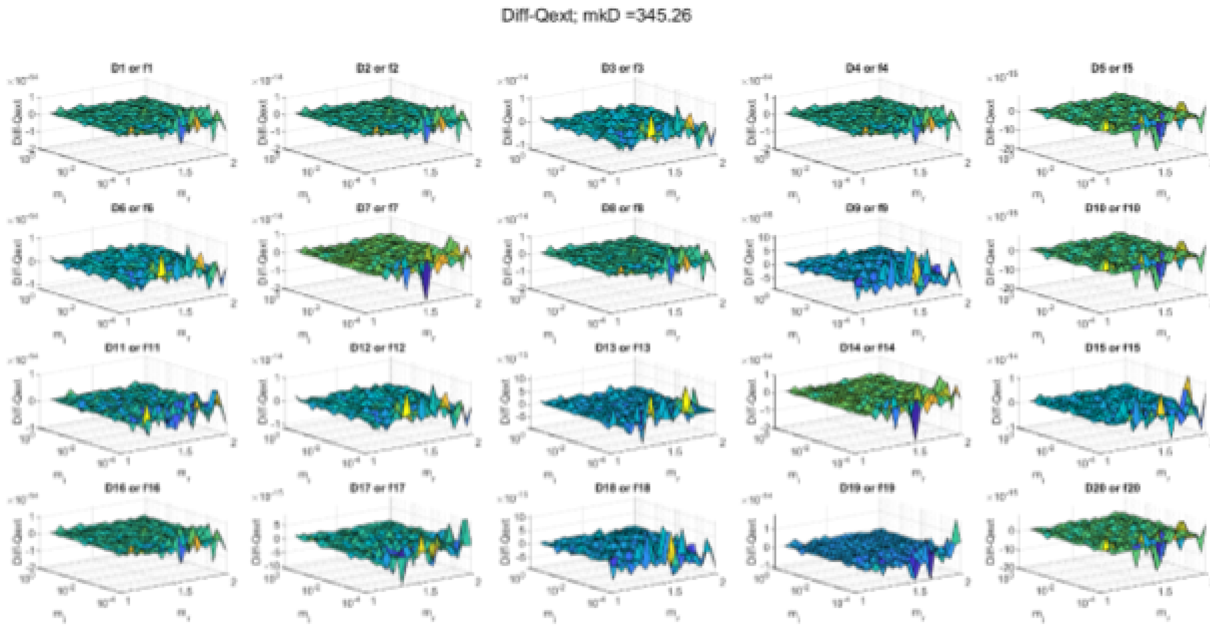


Fig. 3 Differences of $Q_e$ between those in Fig. 1 and those in Fig. 2..

$\sigma_e \equiv \sigma_e(f, r) = \pi r^2 Q_e(f, r)$. If we can parameterize $Q_e$ with $f$ and $r$ as independent variables, it can readily extend to σ and, with a little additional effort, to $\beta_e$ once the DSD is incorporated.

## 3. VERIFICATION OF ξ DEPENDENCE

We first verify that single-scattering quantities, such as $Q_e$ and $Q_s$, are indeed functions of ξ only. We select a set of 20 frequencies, i.e., $f_j$, from 300 to 15 terahertz (TZ) corresponding to $\lambda_j$ from 1 μm to 20 μm at 1μm increment. For each $\xi \equiv mkD$ value in a predetermined set, i.e., $\xi_i$ ={10, 87.37, 164.74, 345.26, 472.63, 500}, we use the frequency set to find $D_l$ and obtain Mie solutions on a grid of $m = m_r + i\, m_i$. That is,

$$D_l = \xi_i/[m(f_j)k(f_j)], \text{ for all } i,\ j$$

where $\xi_i$ and $f_j$ are ξ and $f$ in the respective preselected set. Fig. 1 shows the $Q_e$ surface on the $(m_r, m_i)$ grid for $\xi_i = 345.26$. One can see that the differences among the $Q_e$ surfaces are imperceptible. This is the case for all $\xi_i$.

Next, we select a set of 20 $D_k$ values from 2 μm to 40 μ m at 2 μm increment. Similarly, we find $f_m$ for each $\xi_i$ and obtain Mie solutions on the $(m_r, m_i)$ grid. Fig. 2 shows the $Q_e$ surfaces, again for $\xi_i = 345.26$. Again, the differences among the 20 $Q_e$ surfaces are imperceptible. C

Compared to those in Fig. 1, there are no perceptible differences either. We plot in Fig. 3 the difference between the sets shown in Fig. 1 and Fig. 2. Indeed, the difference is near the precision of the double-precision floating point number used for the numerical Mie solution, with absolute values around $10^{-14}$. This is also versified from all $\xi_i$.

## 4. FITTING $Q_e$

In the last section, we use a grid of $(m_r, m_i)$ to verify the generality of the extinction efficiency's sole dependence on ξ. In this section, we use $m(f)$ for liquid water and obtain Mie solutions for a grid of $(f, D)$. Fig. 4 displays $Q_e$ for water droplets as a function of frequency and droplet diameter, i.e. $Q_e(f, D)$. The blue dots denote the Mie solution for $Q_e$ at $(f_j, D_k)$. It is overlaid with a fitted surface, i.e., parameterized on $(f, D)$. We note that the fit is

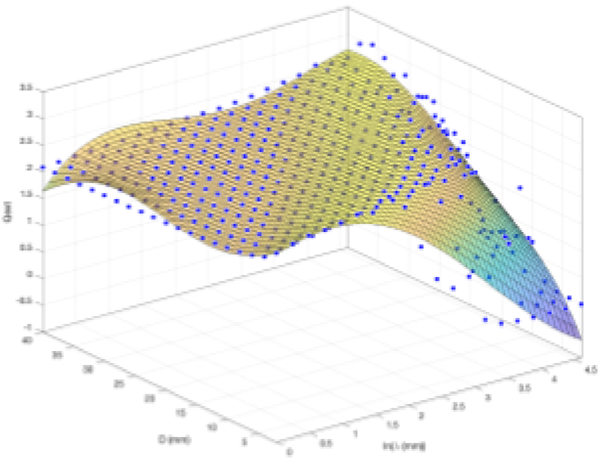

Fig. 4 $Q_e$ as a function of wavelength and droplet diameter with a polynomial surface fit.

generally adequate, except at longer λ and smaller $D$, where there is a rapid change in slope.

Although the polynomial surface fit generated by MatLab is less than ideal, we believe the prospect of achieving better fit is still great. We have several strategies in plan to try out. For example, we plan to adaptively increase the $(f, D)$ grid density at where the $Q_e$ change is rapid, i.e. greater slope. We may also carve the $(f, D)$ into pieces, requiring the $Q_e$ surface to be smooth for each of them, i.e., exploring piecewise parametrization.

## 5. CONCLUSION

We have demonstrated that the extinction efficiency $Q_e$ is solely a function of $\xi = mkD$. In fact, this is the case for all scattering quantities of homogeneous spheres as dictated by the Mie solution. As such, ξ provides a simplified pathway to parameterize the scattering quantities. We have demonstrated a fit for $Q_e$ with frequency $f$ and droplet diameter $D$ as the independent variables. The fit is satisfactory for a significant portion of the $(f, D)$ plane, but not when $f$ is low and $D$ is small where the surface has large slopes. However, we believe the approach shows potential and is worth further pursuit.

## 6. REFERENCES

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